Analytic Hierarchy Process Based Selection of Leaders of Start-up Enterprises

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Abstract: The strength of leadership in the context of start-ups is closely related to the innovation and development of enterprises. The stronger the leadership of enterprises, the better the innovation and development of enterprises. Therefore, it is of great significance to study of the leadership of enterprises. We can cultivate and improve enterprise leadership to meet the innovation and development needs of enterprises in different situations. This paper uses four factors as evaluation indicators of enterprise leadership: strategic, communication, personal and incentive. Assuming the relationship of various influencing factors, it establishes an analytic hierarchy process (AHP) model to select the best leaders among three kinds of enterprise leaders. This offers guidance that contributes to the theory and application of the AHP.

Keywords: leadership; analytic hierarchy process; relative weight vector; consistency inspection.

1. Introduction

The topic of leadership has been extensively studied in science. Many scholars focus on cultivating and enhancing entrepreneurial leadership. According to Ying (2022), the competitiveness of private companies is related to many factors, including cultivating and promoting entrepreneurial leadership, which they discuss at length. LiLi (2021) explains that the leadership of entrepreneurs consists of the ability to learn, teach and make decisions. Her work examines the promotion of leadership from the perspective of culture. This perspective suggests that it is the absence and underdevelopment of corporate management that leads to a lack of improvement in entrepreneurial leadership. Lingbiao (2020) suggests that entrepreneurial leadership in the new era can improve through five aspects, namely: supreme belief, insight into decision-making, undisputed attractiveness, load coordination, and pressure resistance. Some scholars like Donghong and Xiang (2014) take a different perspective. They propose that entrepreneurial leadership is an internal driver for the growth of technology-enabled startups, affecting the strategy, decision-making of entrepreneurial teams, and organizational operations management. It is concluded that entrepreneurial leadership drives the growth of science and technology companies by influencing core team formation, development mechanisms, organizational system, and social network integration. Leadership improves external competitiveness by developing internal innovation, business growth and integration.

Hajazi et al. (2012) have developed a measure of entrepreneurial leadership that consists of four groups of factors: strategic, communicative, personal and motivational. While there are
other strategic factors associated with leadership, this study will focus on these four. From a governance assessment perspective, we collect data from three different types of enterprise leaders, which we use to conduct research and analysis on the four factors mentioned above. Based on the influence of all four factors, the strongest and most comprehensive type of enterprise is selected.

As start-up companies have expanded and matured, their management methods have fundamentally changed. Management systems are gradually moving away from the traditional method of relying solely on individually selected leadership towards the establishment of a more scientific evaluation system to improve order and select the best leadership for an organization. This plays an important role in promoting the management of large companies. According to the survey, two out of three companies surveyed use the AHP for corporate governance in their system. In a mature enterprise management, there are other management methods (such as statistical analysis and data mining, etc.) that affect its operation. However, these management methods have shortcomings as they rely on mathematical deduction skills and certain theorems and assumptions. People-oriented management methods use simple objective descriptions and quantification that are too rigid to be applied in changing situations. However, the AHP is well suited to complex and changing situations and the combination of qualitative and quantitative analysis.

To assess entrepreneurial leadership, we collected questionnaires from employees that focused on strategic, communicative, personal, and incentive factors. We then examined the strengths and weaknesses of different companies, considering the influence of these four factors, and used the AHP to comprehensively assess which company has the strongest leadership using the AHP's comprehensive assessment process. The calculation showed which aspects of leadership need improvement. By providing guidance for the management of start-up companies, this research makes a valuable contribution to their sustainable development from small to large companies.

2. Review of Related Literature

Analytic Hierarchy Process (AHP) is a systematic and hierarchical analysis method which is a combination of qualitative and quantitative analysis and established by T.L. Suaty, an American operational research scientist, in the 1970s. It is an effective method to transform semi-qualitative and semi-quantitative problems into quantitative problems. Its essence is a hierarchical way of thinking, that is, to decompose complex problems into multiple component factors and form a hierarchical structure according to the dominant relationship of these factors. By comparing them layer by layer, the overall ranking of relative importance of decision-making schemes can be determined. It provides quantitative basis for analyzing, making decisions, predicting or controlling the development of things. Therefore, analytic hierarchy process (AHP) is particularly suitable for solving complex problems that are difficult to be completely dealt with by quantitative methods. It has important applications in the fields of comprehensive evaluation, planning, resource allocation, prioritization, decision making and so on.

The thought of analytic hierarchy process (AHP) is basically consistent with people's thinking process of a multi-level, multi-factor and complex decision-making problem, which is characterized by hierarchical comparison and comprehensive optimization.

From our analysis of the collected data, the leadership of start-up companies is closely related to the innovation and development of companies. This is a qualitative and multifactorial optimization problem. Based on this, we assessed the weight of the four factors influencing the leadership of startups, as shown in Table 1
Table 1. Weighting table of four influencing factors

<table>
<thead>
<tr>
<th>Factor</th>
<th>Grade</th>
</tr>
</thead>
<tbody>
<tr>
<td>strategical factor</td>
<td>3</td>
</tr>
<tr>
<td>Communication factors</td>
<td>1</td>
</tr>
<tr>
<td>personal factors</td>
<td>2</td>
</tr>
<tr>
<td>Motivational factors</td>
<td>5</td>
</tr>
</tbody>
</table>

At the same time, we collected samples and calculated the effect of the four influencing factors on the three types of leaders with numerical values from 1 to 9. This analysis identified which types of leaders are better suited for organizational innovation and development, as shown in Table 2.

Table 2. Score of three types of leadership in four influencing factors

<table>
<thead>
<tr>
<th>Type of Entrepreneurial Leader</th>
<th>Strategical Factor</th>
<th>Communication factors</th>
<th>Personal factors</th>
<th>Incentive factors</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_1$</td>
<td>6</td>
<td>4</td>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>$P_2$</td>
<td>8</td>
<td>5</td>
<td>5</td>
<td>3</td>
</tr>
<tr>
<td>$P_3$</td>
<td>5</td>
<td>7</td>
<td>3</td>
<td>4</td>
</tr>
</tbody>
</table>

According to the data collected and sorted above, it is advisable to use the analytical hierarchy process (AHP) to establish a research model. Ping, Y. (2018) AHP is a decision-making method that decomposes the elements always related to decision-making into goals, criteria, schemes and other levels to perform qualitative and quantitative analysis.

This method was proposed by an American operations researcher, Professor Sati from the University of Pittsburgh. In the early 1970s he did research for the US Department of Defense on how the distribution of power in different industrial sectors affected national welfare. He applied network systems theory along with a multi-objective, comprehensive assessment method. With this method, the relative weight vector is determined by considering the pairwise comparison matrix of influencing factors on the route guidance. The relative weight vector then results from the pairwise comparison.

3. Research Methods

The basic steps of using AHP to solve complex decision problems are as follows:

1. Analyze the relationship between various factors in practical problems and establish the hierarchical structure of practical problems. It is generally divided into three layers, the top layer is the objective layer, the middle layer is the criterion layer, and the programme layer is the scheme layer.

2. Construct a pairwise comparison matrix, and make a pairwise comparison of the importance (or influence) of each factor at the same level to a criterion (or object) in the upper layer to determine the comparison matrix.

3. The relative weight of each factor for each criterion is calculated by the comparison matrix, and the consistency test of the comparison matrix is carried out.

4. Calculate the combination weight of the programme layer and the objective layer, and conduct the combination consistency test. Finally, make a comprehensive ranking according to the weight size, and make a decision scheme.

The four steps are as follows:
(1) Establish the hierarchy diagram
The first step to solve a problem with analytic hierarchy process is to analyze the factors involved in the problem and the relationship between each other. The factors are stratified and a tree-like hierarchical structure model is constructed, which is called hierarchical structure diagram. The hierarchy diagram of general problems is divided into three layers: The top layer is the objective layer (O) : there is usually only one goal or desired outcome to solve the problem.
The middle layer is the criterion layer (C) : represents the factor (or standard) that affects (or measures) the target, and each factor is called a criterion. When there are too many criteria (for example, there are more than 9 criteria), the sub-criterion layer should be further decomposed. At this time, the criterion layer of the middle layer can have several sub-layers.
The lowest layer is the programme layer (P), which represents all the solutions and measures to solve the problem.

(2) Construct the comparison matrix
When the analytic hierarchy process is used to deal with complex decision problems, it is necessary to compare the influence degree of each factor at the next level on the related factors at the next level according to the hierarchy diagram constructed in the first step. At this time, it is not to compare all factors together, but to compare factors in the same level in pairs, and use relative scale to measure, as far as possible to avoid the difficulty of comparing factors of different nature. Suppose to compare the influence degree of multiple elements on the upper layer, determine the ratio of influence degree of any two factors, and measure it according to the scale of 1-9. The matrix composed of all the comparison results as elements becomes the comparison matrix (or judgment matrix).

(3) Determine the relative weight vector and consistency test
Generally, there are three methods of characteristic root method, sum method (arithmetic average method) and root method (geometric average method) to calculate and determine the relative weight vector. Generally, the characteristic root method is selected and the mathematical software Matlab is used for calculation. Then the random consistency index is used for consistency test.

(4) Determine the combination weight vector and combination consistency test
Combine the weight of each element in the next layer with the weight of each element in this layer to get the weight vector of each element to the upper layer (the top layer). Then, the above random consistency indexes are used to test the combination consistency. Through the test, the index of the maximum weight vector obtained can be used as the final decision basis.

4. Results and Discussion
4.1. Model Building based on the Hierarchical Analysis Method
4.1.1. Establishing the Hierarchy Diagram
In order to select the best type of entrepreneurial leaders, the decision-making problem is divided into three levels. The upper level is the objective level (selecting corporate leadership). The middle is the criterion layer, including four criteria: strategic factor, communication factor, personal factor and incentive factor (Wanling, 2011). The lower layer is the scheme layer (there are $P_1$, $P_2$ and $P_3$ startup leadership options). Qiyuan, J. et al (2011) The established hierarchy
is shown in Figure 1:

![Heirarchical Diagram](image)

**Figure 1. Heirarchical Diagram**

4.1.2. **Constructing Pairwise Comparison Arrays**

Compare all influencing factors in pairs. Compare the influence of n factors \( C_1, C_2, \ldots, C_n \) in a layer on one factor in the upper layer, take two factors \( C_i \) and \( C_j \) each time, and use \( a_{ij} \) to express the ratio of the influence degree of \( C_i \) and \( C_j \) on \( O \), and all results can use the comparison matrix

\[
A = (a_{ij})_{n \times n}, \quad a_{ij} > 0, \quad a_{ji} = \frac{1}{a_{ij}}
\]

To show that, there must be an \( a_{ii} = 1 \)

4.1.3. **Pairwise Comparison Matrix between Criterion level and Target level**

Using the collected statistical data, use \( C_1, C_2, \ldots, C_4 \) to express the four criteria (strategic, communication, personal factor and incentive) in turn, then the comparison matrix of the influence degree of the four factors in the criteria layer on the objective layer is as follows

\[
A = \begin{bmatrix}
1 & 3 & 3/2 & 3/4 \\
1/3 & 1 & 1/2 & 1/4 \\
2/3 & 2 & 1 & 1/2 \\
4/3 & 4 & 2 & 1
\end{bmatrix}
\]

4.1.4. **Pairwise Comparison Matrix of the Scheme Layer versus the Criterion layer**

Using the collected statistical data, the comparison matrix of four factors in the scheme layer to the criterion layer is:
4.2. The Model’s Solution

4.2.1. Determining the Relative Weight of the Vectors

In general, if a comparison matrix $A$ satisfies

$$a_{ij} \cdot a_{jk} = a_{ik}, \quad i, j, k = 1, 2, \ldots, n$$

Then $A$ is called a consistency matrix, and $A$ has a unique non-zero characteristic root which is $n$. Any column vector of matrix $A$ corresponds to the characteristic vector of the characteristic root $n$. The normalisation of the characteristic vector represents the relative weight vector of factor $C_1, C_2, \ldots, C_n$ to the upper layer factor (Guo, 2018).

4.2.2. Consistency Test

If the paired comparison matrix $A$ is not a consistency matrix, use the eigenvalue method to calculate the relative weight vector and perform consistency check (Zijing, 2018).

1. **Find the characteristic root according to the characteristic polynomial of comparison matrix $A$**

$$|\lambda I - A| = 0$$

Where, $\lambda$ represents the characteristic root of matrix $A$, and $I$ represents the identity matrix. Let's assume that the largest characteristic root of matrix $A$ is $\lambda_{\text{max}}$.

2. **Determination of the relative weight vector**

$$\left(\lambda_{\text{max}} I - A\right) x = 0$$

A solution of the above equation system about $x$ is the eigenvector corresponding to the largest eigenvalue $\lambda_{\text{max}}$ of matrix $A$. If the eigenvector is normalized, the relative weight vector of matrix $A$.

According to the above method, the maximum characteristic root and relative weight

$$B_1 = \begin{bmatrix} 1 & 6/8 & 1 \\ 8/6 & 1 & 8/5 \\ 1 & 5/8 & 1 \end{bmatrix}$$

$$B_2 = \begin{bmatrix} 1 & 4/5 & 4/7 \\ 5/4 & 1 & 5/7 \\ 7/4 & 7/5 & 1 \end{bmatrix}$$

$$B_3 = \begin{bmatrix} 1 & 1/5 & 1/3 \\ 5 & 1 & 3 \\ 3 & 1/3 & 1 \end{bmatrix}$$

$$B_4 = \begin{bmatrix} 1 & 5/3 & 5/4 \\ 3/5 & 1 & 3/4 \\ 4/5 & 4/3 & 1 \end{bmatrix}$$
vector of paired comparison matrix $A$ are calculated by MATLAB programming:
\[
\lambda_{\text{max}} = 4.0062 \\
\omega^{(2)} = (0.2766, 0.0922, 0.1953, 0.4360)^T
\]

It can be seen from the relative weight vector of matrix $A$ that incentive factors account for the largest weight, followed by strategic factors, and communication factors which account for the smallest. This makes sense in light of the practical needs of enterprise development. Similarly, it can be obtained that the maximum eigenvalue and relative weight vector of the four paired comparison matrices $B_1, B_2, B_3, B_4$ of the programme layer to the criterion layer.

**Table 3.** The relative weight vector $\omega_k^{(3)}$ of $B_k$ and the maximum characteristic root $\hat{\lambda}_k$

<table>
<thead>
<tr>
<th>$k$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\omega_k^{(3)}$</td>
<td>0.2979</td>
<td>0.2500</td>
<td>0.1062</td>
<td>0.4167</td>
</tr>
<tr>
<td></td>
<td>0.4218</td>
<td>0.3125</td>
<td>0.6333</td>
<td>0.2500</td>
</tr>
<tr>
<td></td>
<td>0.2803</td>
<td>0.4375</td>
<td>0.2605</td>
<td>0.3333</td>
</tr>
<tr>
<td>$\hat{\lambda}_k$</td>
<td>3.0037</td>
<td>3</td>
<td>3.0387</td>
<td>3</td>
</tr>
</tbody>
</table>

**3) Conformance test of the pairwise comparison matrix**

Most pairwise comparison matrices of order 3 and above are not consistency matrices. In fact, as long as the degree of inconsistency is within a certain allowable range, it is considered that the constructed comparison matrix is appropriate. According to the method given by Satty, when $n \geq 3$, the ratio of the consistency index $CI$ of the n-order comparison matrix to the random consistency index $RI$ is the consistency ratio $CR$, that is:

\[
CR = \frac{CI}{RI}
\]

According to the above method, the consistency index and consistency ratio of paired comparison matrix $A$ are calculated by MATLAB programming:

\[
CI^{(2)} = 0.0021, \quad CR^{(2)} = \frac{CI^{(2)}}{RI_4} = 0.0023
\]

Because $CR^{(2)} = 0.0023 < 0.1$, it can be considered that the inconsistency degree of $A$ is within the allowable range, and its eigenvector can be used as its relative weight vector.

Similarly, the consistency index and consistency ratio of the four pairwise comparison matrices $B_1, B_2, B_3, B_4$ of the scheme layer to the criterion layer can be obtained as follows:

**Table 4.** Consistency index $CI_k^{(3)}$ and consistency ratio index $CR_k^{(3)}$ of $B_k$

<table>
<thead>
<tr>
<th></th>
<th>$B_1$</th>
<th>$B_2$</th>
<th>$B_3$</th>
<th>$B_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$CI_k^{(3)}$</td>
<td>0.0019</td>
<td>0</td>
<td>0.0193</td>
<td>0</td>
</tr>
<tr>
<td>$CR_k^{(3)}$</td>
<td>0.0033</td>
<td>0</td>
<td>0.0333</td>
<td>0</td>
</tr>
</tbody>
</table>

It can be seen from table that the four pairwise comparison matrices of the programme layer and the criterion layer have passed the consistency test, indicating that the corresponding feature vector can be used as the relative weight vector.
4.2.3. **Determining the Combined Weight Vector**

The above results have obtained the relative weight vector $\omega^{(2)}$ of the criterion layer to the objective layer and the relative weight vector $\omega^{(3)}_k (k = 1, 2, \ldots, 4)$ of the programme layer to each criterion. What affects the decision is the weight vector of the scheme layer to the target layer, which is called the combined weight vector, which can be recorded as $\omega^\ast$.

For the above three levels of decision-making problems, there is only one factor in the first layer, and there are multiple factors in the second and third layers. The relative weight vector of the second layer to the first layer is $\omega^{(2)}$, and the relative weight vector of the five criteria of the third layer to the second layer is $\omega^{(3)}_k (k = 1, 2, \ldots, 4)$ respectively, and $\omega^{(3)}_k$ is used as the column vector to form the matrix:

$$
\omega^{(3)} = [\omega^{(3)}_1, \omega^{(3)}_2, \ldots, \omega^{(3)}_5]
$$

Then the combined weight vector of layer 3 to layer 1 is

$$
\omega^\ast = \omega^{(3)} \omega^{(2)} = (0.3078, 0.3781, 0.3141)
$$

According to the results of the above calculation vector, the weight of scheme $P_2$ is larger than $P_1, P_2$, which should be the best choice for start-ups with relatively strong leadership.

4.2.4. **Combination Consistency Test**

For the above decision-making problems, the combination consistency check should be carried out layer by layer. There is only one factor in layer 1, multiple factors in layers 2 and 3, the consistency ratio of layer 2 is $CR^{(2)}$, and the consistency index of layer 3 to the four factors in layer 2 is $CI^{(3)}_k (k = 1, 2, \ldots, 4)$ respectively.

Then the consistency index of layer 3 combination is

$$
CI^{(3)} = [CI^{(3)}_1, CI^{(3)}_2, \ldots, CI^{(3)}_4] \omega^{(2)} = 0.0043,
$$

Then the combined random consistency index of layer 3 is

$$
RI^{(3)} = [RI^{(3)}_1, RI^{(3)}_2, \ldots, RI^{(3)}_4] \omega^{(2)} ,
$$

The layer 3 combination consistency ratio is

$$
CR^{(3)} = \frac{CI^{(3)}}{RI^{(3)}} = 0.0074 ,
$$

The combined consistency ratio of layer 3 to layer 1 is

$$
CR^\ast = CR^{(3)} + CR^{(2)} = 0.0074 + 0.0023 = 0.0097 ,
$$

Because $CR^\ast < 0.1$, the combination consistency test passes, then the combination weight vector $\omega^\ast$ can be used as the basis for the final decision, and $P_2$ can be selected as the best entrepreneur leadership.

We also investigated the innovation development of this category of start-ups in real-world cases over recent years, and found that the development of the enterprises of category
was indeed better than that of other enterprises. This further verified our calculation. Therefore, entrepreneurial leadership is closely related to the innovative development of enterprises.

This research has practical application as it can be used to help business decision makers choose the best leaders manifesting the important factors that are vital to organization. Decision makers must consider many factors and evaluation criteria before making a final decision. For example, if you choose a company with strong leadership, you can choose one of A, B, and C as your ideal company. The selection should take into account the strategic, communicative, personal and incentive factors that influence the company. These factors limit and influence each other. We call such a complex system a decision system. The comparison of many factors in these decision systems cannot be described quantitatively. Therefore, semi-qualitative and semi-quantitative problems must be transformed into quantitative computational problems.

5. Conclusion

The paper posits that AHP can be an alternative in effectively solving complex decision problem of choosing a leader that manifests the factors essential to the organization. It provides a quantitative basis for analysis and final decision making by comparing the importance of various related factors layer by layer. AHP is based on an in-depth analysis of the nature of the influencing factors and the internal relationships of complex decision problems. It uses less quantitative information and more mathematics to simplify complex decision problems with multiple objectives and multiple criteria or unstructured features. It is particularly useful when it is difficult to measure decision outcomes directly and accurately.

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